C.2. 微積分期中考 (2003 Spring) 系級: 學號: 姓名: P1

1. Use power series to solve the following initial value problem: $y' - y = x$, $y(0) = 1$. (15分)

2. Find $\lim_{n\to\infty}(1 + \frac{1}{n})^{-n}$. (10分)

3. Use power series to evaluate $\lim_{x\to 0} \frac{1-\cos^2 x}{\ln(1-x)+\sin x}$. (15分)

4. Show that the Maclaurin series for $\cos x$ converges to $\cos x$ for every value of x. (15分)

C.2 5. Identify the function $f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$, $-1 < x < 1$ (15分) P2

6. Find the following series' radius and interval of convergence (10分)

$$\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2}.$$

7. Show that

$$\int_2^{\infty} \frac{1}{x(\ln x)^p} dx \quad (p \text{ a positive constant})$$

converges if and only if $p > 1$. (10分)

8. Use the Integral Test to determine whether the following series converges or diverges (10分)

$$\sum_{n=1}^{\infty} \frac{e^n}{1 + e^{2n}}.$$